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NUMERICAL METHODS IN LOAD FLOW ANALYSIS: AN APPLICATION TO THE NIGERIAN GRID SYSTEM

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ABSTRACT

Two primary considerations in the development of an effective engineering computer program are the formulation of a mathematical description of the problem and the application of a numerical method for a solution. The mathematical formulation of the load flow study results in system of algebraic nonlinear equations, and owing to this the numerical solution is reached by iteration. The different mathematical techniques used for load flow study are Gauss-Seidel, Newton-Raphson, Decoupled and Stott’s fast decoupled methods. This paper presents the results of evaluation of study for running power flow program based on Gauss-Seidel, Newton-Raphson and Fast decoupled algorithms. Three tested system IEEE 5-Bus, IEEE 30-Bus and the Nigerian 28-Bus electrical power system are considered using the three numerical solutions. The numerical result of running power flow studies for IEEE 5-Bus, 30-Bus and the Nigerian 28-Bus systems are presented and comparatively discussed.

KEYWORDS: Fast Decoupled Gauss, Seidel, Iteration, Load Flow, Newton Raphson, Numerical Solution, Power Flow

INTRODUCTION

The state of a power system and the methods of calculating this state are extremely important in evaluating the operation of the power system, control of this system and the determination of future expansion for the power system (Gilbert et al, 1998). Numerical methods are techniques by which mathematical problems are formulated so that they can be solved with arithmetic operations and they usually provide only approximate solution. To facilitate the development of numerical problems, the need to set up procedures this leads to algorithms. The solution however involves the following steps; modeling, choice of numerical methods, operation, result and interpretation of results. In power systems, power are known rather than currents; thus, the resulting equations in term of power, known as the power flow equations become non-linear and must be solved by iterative techniques using numerical methods (Saadat, 2006).

In its basic form the load flow problem involves solving a set of nonlinear algebraic equations which reperesent the network under steady-state conditions (Acha et al, 1996). Owing to the nonlinear nature of the power flow equations, the numerical solution is reached by iteration (Acha, etal 2004). Steady state of power systems may be determined by solving the power flow equations that mathematically are represented by a set of non-linear algebraic equations. During last three decades, various methods of numerical analysis for solving a set of non-linear algebraic equations have been applied in solving load flow analysis problems [1-7].The most commonly used iterative methods are the Gauss-Seidel, the Newton-Raphson and Fast Decoupled method (Kothari and Nagrath, 2008).

The desirable features to compare the different power flow methods can be the speed of solution, memory storage requirement, and accuracy of solution and the reliability of convergence depending on a given situation. (Keyhani, et al 1989). In the load-flow analysis, its main objective is the calculation of all bus voltages magnitudes and angles, and consequently the power flows over the transmission lines (Ravi Kumar and Nagaraju, 2007). Test cases were carried out on IEEE 5-Bus, IEEE 30-Bus and Nigerian 28-Bus grid system. In this work a power flow analysis was carried out on the tested electrical power system using the three numerical solution methods.

OAD FLOW SOLUTION TECHNIQUES

The load flow analysis, in power system parlance, is the steady state solution of the power system network. (Kothari and Nagrath, 2008). The power flow problem involves determining voltages and line flows, in a large sparse electrical network, for a given load and generation schedule (Vader et al, 1999). In this analysis, the power system network is modeled as an electric network and solved for the steady state power, voltages at various buses and hence the power at the slack bus and power flows through inter connecting power channels (Ravi Kumar and Silla Nagaraju, 2007). Four quantities which are associated with each bus are voltage magnitude $|v|$, phase angle δ, real power P, and reactive power Q. The system buses are generally classified into three types namely: Slack bus, Load buses and Generator buses where two variables are specified and others two to be determined (Saadat, 2006).

Gauss-Seidel Method

The Gauss-Seidel method is an iterative algorithm for solving a set of non-linear algebraic equations. It was one of the methods used in load flow studies. Here a solution vector is assumed and one of the equations is used to obtain the revised value of a particular variable. The solution vector is immediately updated in respect of this variable. The process is then repeated for all the variables thereby completing one iteration. The iterative process is then repeated till the solution vector converges within prescribed accuracy. The convergence is quite sensitive to the starting values assumed (Nagrath and Kothari, 2006).

Newton-Raphson Method

The origin of the formulation of the power flow problems and the solution based on Newton-Raphson's technique dates back to the late 1960s (Federico, 2008). It is an iterative method which approximates a set of non-linear simultaneous equations to a set of linear simultaneous equations using Taylor's series expansion and the terms are limited to the first approximation (Wadhwa, 1991). Its convergence characteristics are relatively more powerful compared to other alternative processes and the reliability of Newton-Raphson approach is comparatively good, since it can solve cases that lead to divergence with other popular processes.(Ravi Kumar and Silla Nagaraju, 2007).

Fast-Decoupled Method

It was demonstrated in the late 1970s that the storage and computing requirements of the Newton-Raphson method could be reduced very significantly by introducing a series of well-sustained simplifying assumptions. These assumptions are based on physical properties exhibited by electrical power system, in particular in high-voltage transmission system. The resulting formulation is no longer a Newton-Raphson method but a derived formulation described as Fast-decouple method. The power mismatch equations of both methods are identical but their Jacobians are quite different; the Jacobians elements of the Newton –Raphson are voltage –dependent whereas those of the Fast decouple method are voltage-independent (i.e. constant parameters). Moreover, the number of Jacobian entries used in the Fast-decouple method is only half of those used in the Newton-Raphson method but has strong convergence characteristics. However, an asset of the Fast decouple Newton- Raphson method is the fact that one of its iterations takes only a fraction of the time required by Newton-Raphson's method of iterations (Acha, et al 2004).

POWER FLOW EQUATIONS

The current I at bus i is given as

$$I_i = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j \quad j \neq i \tag{1}$$

While the powers balance equation for a power system network at bus ί can be written as;

$$S_i = P_i + jQ_i = V_i I_i^* \tag{2}$$

Or

$$I_i = \frac{P_i - jQ_i}{V_i^*} \tag{3}$$

Using equation (2) in equation (1) gives

$$I_i = \frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j \qquad j \neq i \tag{4}$$

From equation (4), the mathematical formulation of the power flow problem results in a system of algebraic non- linear equations which must be solved by iterative techniques.

Gauss – Seidel Power Flow Solution

In the power flow study, it is necessary to solve the set of non – linear equations represented by equation (4). In the Gauss – Seidel method equation (4) is solved for V_i and the iterative sequence becomes,

$$V^{(k+1)} = \frac{\dfrac{P^{sch} - jQ_i^{sch}}{V_i^{*(k)}} + \sum y_{ij} V_j^{(k)}}{\sum y_{ij}} \qquad j \neq i \tag{5}$$

In writing the KCL, current entering bus i was assumed positive. Thus, for buses where real and reactive powers are injected into the bus, such as generator buses, P^{sch} and Q^{sch} have positive values. For load buses where real and reactive powers are flowing away from the bus, P^{sch} and Q^{sch} have negative values. If equation (4) is solved for P_i and Q_i, we have

$$P_i^{(k+1)} = \mathrm{Re}\,al\left[V_i^{*(k)} \left\{ V_i^{(k)} \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_i^{(k)} \right\} \right] \qquad j \neq i \tag{6}$$

$$Q_i^{(k+1)} = \mathrm{Im}aginary\left[V_i^{*(k)}\left\{V_i^{(k)}\sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_i^{(k)}\right\}\right] \quad j \neq i \tag{7}$$

The bus admittance matrix Y_{bus} is an important network description of the interconnected power system and the power flow equation is usually expressed in terms of the elements of the bus admittance matrix, Y_{bus}. Since the off-diagonal elements of the bus admittance matrix Y_{bus} are $Y_{ij} = -y_{ij}$ and the diagonal elements are $Y_{ii} = \sum yij$ then (5) above becomes,

$$V_i^{(k+1} = \frac{\dfrac{Pi^{sch} - jQi^{sch}}{V_i^{*(k)}} - \sum_{j=i}^{n} y_{ij}V_j^{(k)}}{Y_{ii}} \tag{8a}$$

$$Y_{ij} = Gij - Bij \tag{8b}$$

And,

$$P_i^{(k+!)} = \mathrm{Re}\,al\left(V_i^{*(k)}\left\{V_i^{(k)}Y_{ii} + \sum_{i=1,\, j\neq i}^{n} y_{ij}V_j^{(k)}\right\}\right) \quad j \neq i \tag{9}$$

$$Q_i^{(k+1)} = \mathrm{Im}aginary\left(V_i^{*(k)}\left\{V_i^{(k)}Yii + \sum_{i=1,\, j\neq i}^{n} y_{ij}V_j^{(k)}\right\}\right) \quad j \neq i \tag{10}$$

Y_{ii} includes the admittance to ground of line charging susceptance and any other fixed admittance to ground.

Newton –Raphson Method

The current entering bus i written in terms of the bus admittance matrix as.

$$I_i = \sum_{j=1}^{n} YijVj \tag{11}$$

Expressing this equation in polar form, we have:

$$I_i = \sum_{j=1}^{n} \left|Y_{ij}\right|\left|V_i\right| < \left(\theta_{ij} + \delta_j\right) \tag{12}$$

The complex power at bus i is,

$$P_i - jQ_i = V_i^* I_i \tag{13}$$

Substituting from (12) for I_i in (13)

$$P_i - jQ_i = \left|V_i\right| < -\delta_i \sum_{j=1}^{n} \left|Y_{ij}\right|\left|V_j\right| < \delta_{ij} + \delta_j \tag{14}$$

Separating the real and imaginary parts

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \tag{15a}$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_i - \delta_i + \delta_j) \tag{15b}$$

The load flow equations given by equations (15a) and (15b) can be expanded into Taylor series and the following first order approximations can be obtained:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \tag{16}$$

Here element J_1, J_2, J_3, J_4 are elements of Jacobian matrix.For voltage controlled buses, the voltage magnitudes are known. Therefore, if m buses of the system are voltage controlled, m equations involving ΔQ and ΔV and the corresponding column of the Jacobian matrix are eliminated. Accordingly, there are n-1 power constraints and n-1-m reactive power constraints, and the Jacobian matrix is of order $(2n-2-m)\times(2n-2-m)$. J_1 is of the order $(n-1)\times(n-1)$,J_2 is of the order $(n-1)\times(n-1-m)$, J_3 is of the order $(n-1-m)\times(n-1)$, and J_4 is of the order $(n-1-m)\times(n-1-m)$, where n is load bus and m is generator bus.

The diagonal and off diagonal element of J_1 are:

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j\neq 1} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \tag{17}$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{18}$$

The diagonal and off diagonal elements of J_2 are:

$$\frac{\partial P_i}{\partial |Vi|} = 2|V_i||Y_{ij}|\cos\theta ii + \sum_{j\neq 1} |V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \tag{19}$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{20}$$

The diagonal and off diagonal elements of J_3 are:

$$\frac{\partial Q_i}{\partial \delta_j} = \sum_{j\neq 1} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \tag{21}$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq 1 \tag{22}$$

The diagonal and off diagonal element of J_4 are:

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Yii|\sin\theta_{ij} + \sum_{j\neq 1}|Vj||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \tag{23}$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \quad j \neq 1 \tag{24}$$

The terms $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are the differences between the scheduled and calculated values, known as the power residuals given by:

$$\Delta P_i^{(k)} = P^{sch} - P_i^{(k)} \tag{25}$$

$$\Delta Q_i^{(k)} = Q^{sch} - Q_i^{(k)} \tag{26}$$

The new estimates for bus voltages are:

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)} \tag{27}$$

$$|V_i^{(k+1)}| = |V_i^k| + \Delta|V_i^k| \tag{28}$$

Fast -Decouple Newton- Raphson Power Flow Solution

This is an extension of Newton – Raphson method formulated in polar coordinates with certain approximation which results in fast algorithm for load flow solution. Because power system transmission lines have a very high X/R ratio thus it is reasonably assumed that real power changes (ΔP) are less sensitive to changes in voltage magnitude and are mainly sensitive to changes in phase angle (Δδ). Similarly, the reactive power is less sensitive to changes in phase angle Δδ but mainly sensitive to changes in voltage magnitude. With these assumptions, equation (16) reduces to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \tag{29}$$

or $\Delta P = J_1 \Delta\delta = \frac{|\partial P|}{\partial \delta}\Delta\delta$ (30)

$$\Delta Q = J_4 \Delta|V| = \frac{|\partial Q|}{\partial |V|}\Delta|V| \tag{31}$$

RESULTS AND DISCUSSIONS

In this work, test cases are carried out on IEEE 5-bus, IEEE 30-bus and Nigerian 28 –bus electrical power systems network. In the investigation, a power flow analysis was carried out on the tested electrical power system network using three numerical solution methods of the three iterative techniques algorithms. The power flow analysis of the test cases was implemented using Matlab programming techniques on the three algorithms. The performance of these formulations on a personal computer is comparatively discussed by using the results of the actual implementation.

Computing Time Requirement

Table 1: Computing Time (Seconds) Requirement for the Algorithms

Test Systems	Gauss-Seidel	Newton Raphson	Fast Decouple
IEEE 5 Bus	0.0156002	0.0312002	0.0156001
IEEE 30 Bus	0.0312002	0.0468003	0.0124801
Nigerian 28 Bus	0.0780005	0.0936006	0.0780000

Considering the table 1 above, it shows that the Gauss-Seidel method has the shortest computing time and the overall time for Gauss-Seidel method is longer than fast decoupled method because of the successive number of Gauss-Seidel iterations. The Newton Raphson method result in similar computing time which is much longer than those of Fast decoupled and Gauss-Seidel methods. Shown in Figures 1, 3 are the bar chart showing the three algorithms with the computing time.

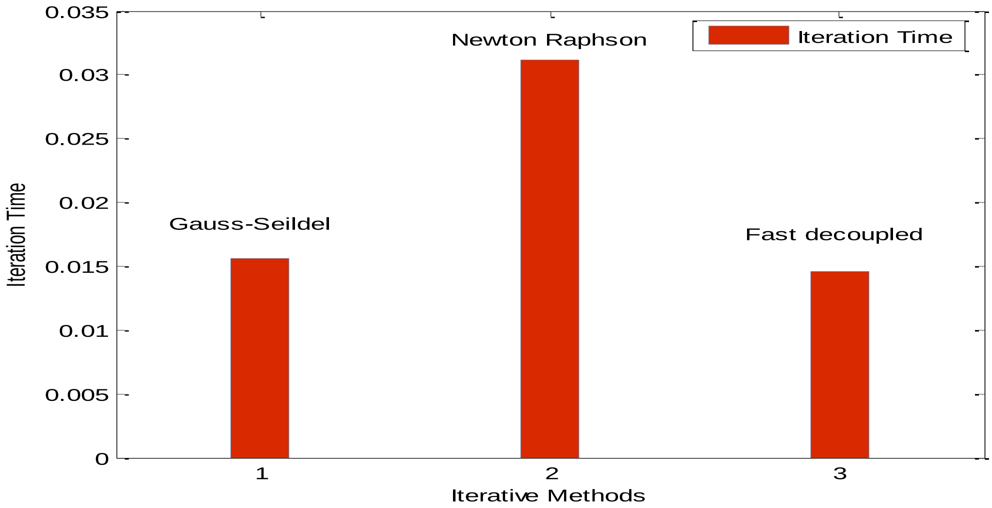

Figure 1: The Iteration Time and Methods for 5 Bus Systems

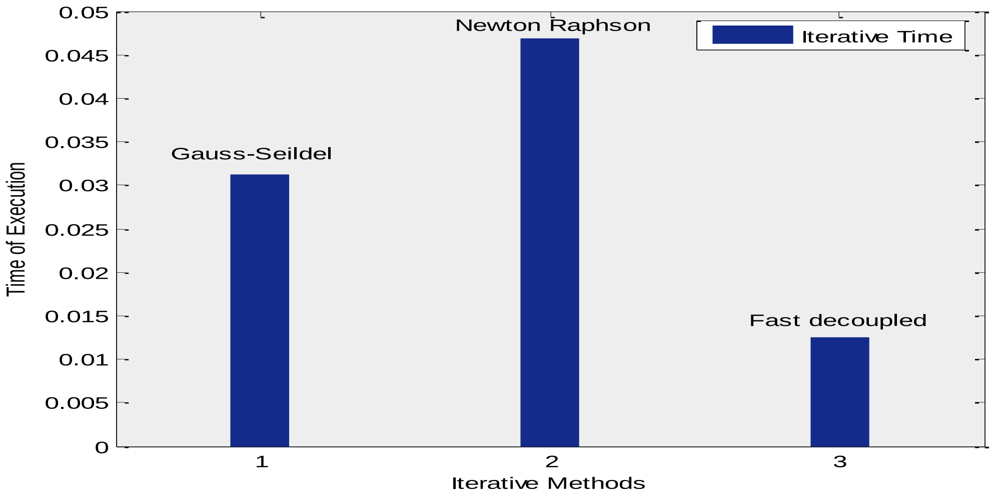

Figure 2: The Iteration Time and Methods for 30 Bus Systems

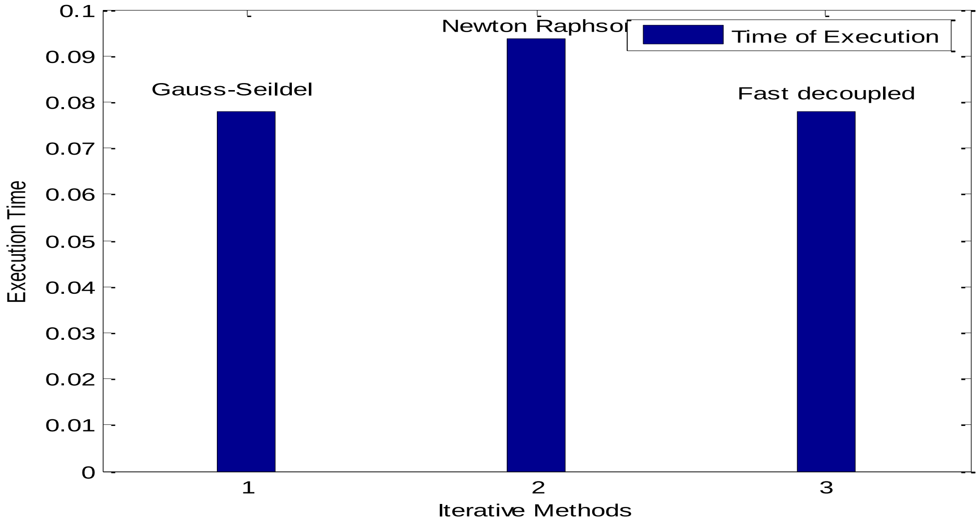

Figure 3: The Iteration Time and Methods for 28- Bus Systems

Iteration Requirements

As seen from Table 2 the Gauss-Seidel method needs a larger number of iterations to converge to a given power mismatch tolerance, compared to the other two methods. The Fast decouple method needs more iterations to converge than the Newton Raphson. But as indicated in the last discussion, the computing time requirement for Fast decouple method per iteration is much less than the Newton Raphson method Shown in Figure 4, 6 is the bar chart showing the three algorithms with the necessary number of iterations.

Table 2: Iteration Number

Test Systems	Gauss-Seidel	Newton Raphson	Fast Decoupled
IEEE 5 bus	12	3	6
IEEE 30 bus	34	4	5
Nigerian 28 bus	69	4	5

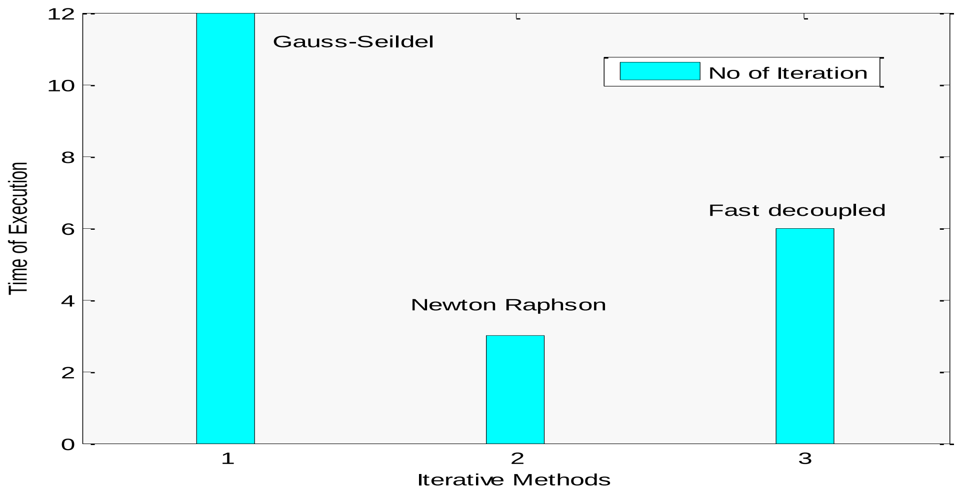

Figure 4: The Number of Iterations Obtained with the 5 Bus Systems

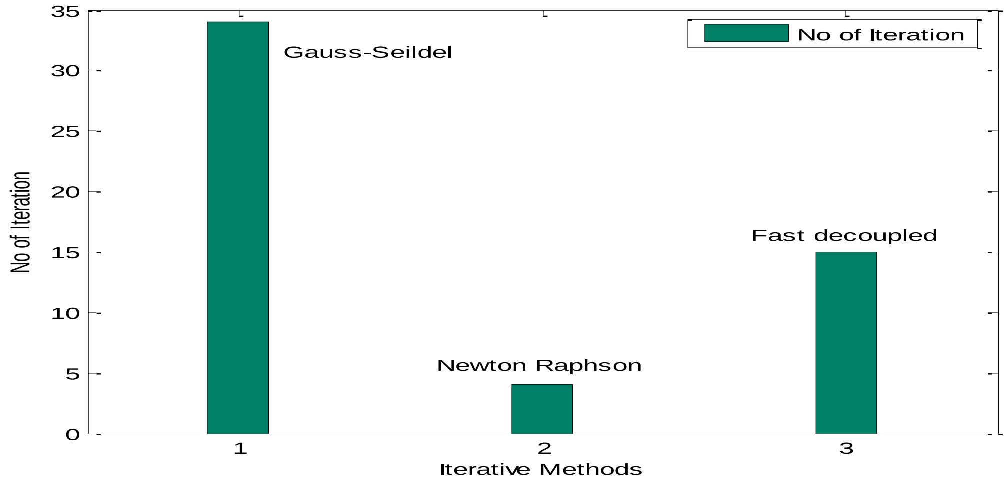

Figure 5: The Number of Iterations Obtained with the 30-Bus Systems

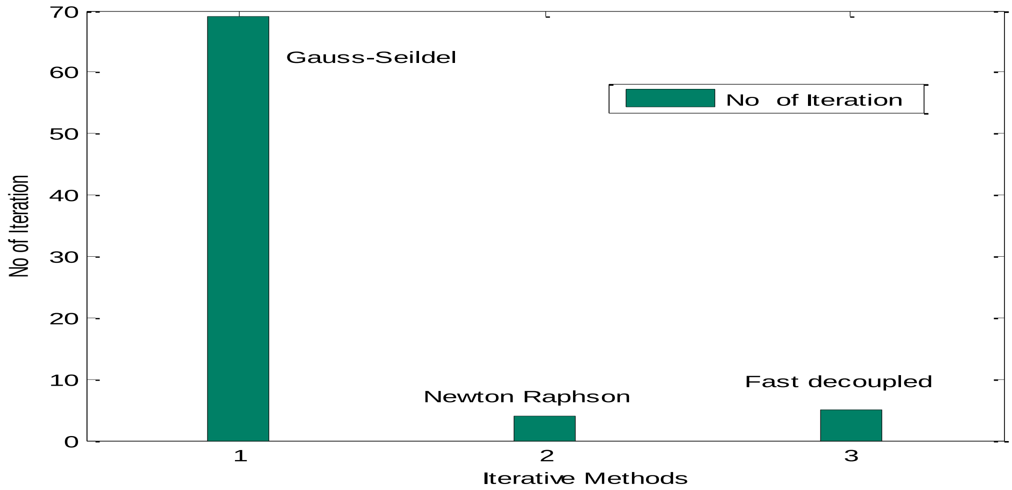

Figure 6: The Number of Iterations Obtained with the 28-Bus Systems

CONCLUSIONS

A performance evaluation for running power flow programs is presented. Three different algorithms namely Gauss- Seidel, Newton Raphson and Fast Decoupled are implemented and case studies are carried out for 5, 30 IEEE-Buses and Nigerian 28-Bus test power systems. The data gathered via simulation are analyzed with respect to computing time and iteration requirements. The well known properties of these algorithms are confirmed by the case study results. The Newton Raphson method is more reliable because it converges faster with quadratic convergence characteristics and with the least number of iterations, is independent of the system size whereas the Gauss-Seidel is slower, the number of iteration increases directly as the number of the buses of the network. However, since the convergence characteristics of the Fast decouple method is geometric compare to the quadratic convergence of the Newton Raphson, thus it has more number of iteration. In general the Newton Raphson algorithm takes the least number of iteration to converge despite its longer computing time.

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